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Abstracts of Papers to Appear in Future Issues

DISCRETIZATION OF FREE SURFACE FLOWS AND OTHER MOVING BOUNDARY PROBLEMS. K. N. Christodoulou and L. E. Scriven. *University of Minnesota, Minneapolis, Minnesota 55455, USA.*

A system of elliptic partial differential equations and boundary conditions has been developed for generating boundary-fitted finite element discretizations of two-dimensional free and moving boundary problems. Terms in the differential equations are scaled for dimensional homogeneity and adjustable weighting of orthogonality, smoothness, and concentration of the coordinate mesh they govern. Grid points become finite element nodes mapped isoparametrically or subparametrically from a simple or patched computational domain. Concentration terms contain control functions and parameters that influence node spacing along each coordinate independently; overall control is by patchwise parameters and functions. Successful selection of these to follow deforming flow regions is straightforward and is illustrated by analysis of steady and transient slide coating flows.

ON THE ERRORS INCURRED CALCULATING DERIVATIVES USING CHEBYSHEV POLYNOMIALS. Kenneth S. Breuer and Richard M. Everson. *Brown University, Providence, Rhode Island 02912, USA.*

The severe errors associated with the computation of derivatives of functions approximated by Chebyshev polynomials are investigated. When using standard Chebyshev transform methods, it is found that the maximum error in the computed first derivative grows as N^2, where $N+1$ is the number of Chebyshev polynomials used to approximate the function. The source of the error is found to be magnification of roundoff error by the recursion equation, which links coefficients of a function to those of its derivative. Tight coupling between coefficients enables propagation of errors from high-frequency to low-frequency modes. Matrix multiplication techniques exhibit errors of the same order of magnitude. However, standard methods for computing the matrix elements are shown to be ill-conditioned and to magnify methods, the errors are found to be most severe near the boundaries of the domain, where they grow as $(1-x^2)^{-1/2}$ as x approaches ± 1. Comparisons are made with the errors associated with derivatives of functions approximated by Fourier series, in which case it is reported that the errors only grow linearly with N and are evenly distributed throughout the domain. A method for reducing the error is discussed.

NUMERICAL EVALUATION OF AIRY FUNCTIONS WITH COMPLEX ARGUMENTS. R. M. Corless, D. J. Jeffrey, and H. Rasmussen. *University of Western Ontario, London, Ontario, Canada.*

We present two methods for the evaluation of Airy functions of complex argument. The first method is accurate to any desired precision but is slow and unsuitable for fixed-precision languages. The second method is accurate to double precision (12 digits) and is suitable for programming in a fixed-precision language such as FORTRAN. The first method uses the symbolic manipulation language Maple to evaluate either the Taylor series expansion or an asymptotic expansion of each function. The second method extends an idea of J. C. P. Miller to the complex plane. It uses the first method to obtain a grid of points in the complex plane where the functions are known to high precision and then uses Taylor series from these base points. The resulting algorithm is accurate and efficient.

VORTICITY ERRORS IN MULTIDIMENSIONAL LAGRANGIAN CODES. John K. Dukowicz. *Los Alamos National Laboratory, University of California, Los Alamos, New Mexico 87545, USA*; Bertrand J. A. Meltz. *Centre d'Etudes de Limeil-Valenton, B.P. 27, 94195 Villeneuve Saint-Georges Cedex, France.*

We investigate the apparent paradox, as exemplified by the well-known Saltzman test problem, of multidimensional lagrangian codes experiencing mesh tangling when computing one-dimensional irrotational flows. We demonstrate that the cause is the generation of spurious vorticity, or vorticity error, by a nonuniform mesh. Based on this, we investigate two methods of constructing improved lagrangian vertex velocities by removing, or filtering out, this spurious vorticity, rather than by the more common practice of introducing artificial viscosity. The first method reconstructs the velocity from the known flow divergence and from the true vorticity computed by means of a transport equation. The second method, which is much simpler and more efficient, subtracts a divergence-free correction from the velocity, such that the resulting velocity possesses the correct vorticity. We then successfully apply this method to solve a two-dimensional shock refraction problem, a problem which exhibits nonzero intrinsic vorticity.

SIMULATION OF ROLLUP AND MIXING IN RAYLEIGH–TAYLOR FLOW USING THE TRANSPORT-ELEMENT METHOD. Anantha Krishnan and Ahmed F. Ghoniem. *Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA.*

The vortex method is extended to obtain solutions of the variable density vorticity transport equation in cases when vorticity is generated by the action of gravitational body forces as well as inertial baroclinic effects. The convection of a scalar, in this case density, is simulated using the transport-element method. Similar to the vortex method, this is a grid-free, Lagrangian field method in which scalar gradients are transported along particle trajectories while being modified according to the distortion of the flow map. Results are obtained for a Rayleigh–Taylor flow evolving by the action of gravity on a finite temperature gradient. The numerical solution is validated by comparing the growth rate of small perturbations to the results of the linear stability analysis of this flow. Numerical solutions within the nonlinear range are analyzed to study the effect of density ratio on the rollup of the vorticity layer and the mixing which follows this process.

ABSORBING BOUNDARY CONDITIONS FOR FREE SURFACE WAVES. J. E. Romate. *Delft Hydraulics, P.O. Box 152, 8300 AD Emmeloord, The Netherlands.*

In this paper the use of absorbing boundary conditions is investigated for the numerical simulation of gravity waves on an incompressible, inviscid fluid in three dimensions. A review of existing methods is given for

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linear and nonlinear waves, after which first- and second-order partial differential equations are introduced as absorbing boundary conditions for the linearized model. Well-posedness is investigated and it is shown that the reflection properties of the second-order equation are superior to those of the first-order equation.

Absorbing Boundary Conditions for Free Surface Wave Simulations with a Panel Method. J. Broeze and J. E. Romate. *Delft Hydraulics, P.O. Box 152, 8300 AD Emmeloord, The Netherlands.*

The numerical implementation and stability of first- and second-order absorbing boundary conditions for simulating free surface gravity waves are considered. The free surface waves are solved with a panel method. The stability of the boundary conditions is proved, assuming certain properties of the integral operator. Arguments are given that support these assumptions. The theoretical results are confirmed in the test cases and the boundary conditions give low reflections for plane waves.

Upwind Relaxation Methods for the Navier–Stokes Equations Using Inner Iterations. Arthur C. Taylor III. *Old Dominion University, Norfolk, Virginia 23529-0247, USA*; Wing-Fai Ng and Robert W. Walters. *Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061-0238, USA.*

An upwind line relaxation algorithm for the Navier–Stokes equations which employs inner iterations is applied to a supersonic and a subsonic test problem. The purpose of using inner iterations is to accelerate the convergence to steady-state solutions, thereby reducing the overall CPU time. A convergence criterion is developed to assist in automating the inner iterative procedure. The ability of the line inner iterative procedure to mimic the quadratic convergence of the direct solver method is confirmed in both test problems, but some of the non-quadratic inner iterative results were more efficient than the quadratic results. In the supersonic test case, the use of inner iterations was very efficient in reducing the residual to machine zero. For this test problem, the inner iteration method required only about 65% of the CPU time which was required by the most efficient line relaxation method without inner iterations. In the subsonic test case, poor matrix conditioning forced the use of under-relaxation in order to obtain convergence of the inner iterations, resulting in an overall method which was less efficient than line relaxation methods which employ a more conventional CPU savings strategy.

An Improved Upwind Finite Volume Relaxation Method for High Speed Viscous Flows. Arthur C. Taylor III. *Old Dominion University, Norfolk, Virginia 23529-0247, USA*; Wing Fai Ng and Robert W. Walters. *Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061-0238, USA.*

An improved upwind relaxation algorithm for the Navier–Stokes equations is presented, and results are given from the application of the method to two test problems, including (1) a shock/boundary layer interaction on a flat plate ($M_\infty = 2.0$) and (2) a high-speed inlet ($M_\infty = 5.0$). The technique is restricted to high-speed (i.e., supersonic/hypersonic) viscous flows. The new algorithm depends on a partitioning of the global domain into regions or sub-domains, where a relatively thin "elliptic" region is identified near each solid wall boundary, and the remainder of the flowfield is identified to be a single larger "hyperbolic" (i.e., hyperbolic/parabolic in the streamwise direction) region. A direct solution procedure by lower/upper factorization is applied to the elliptic region(s), the results of which are then coupled to a standard line Gauss–Seidel relaxation sweep across the entire domain in the primary flow direction. In the first test problem, the new algorithm reduced total run times as much as 75% when compared to the standard alternating forward/backward vertical line Gauss–Seidel (VLGS) algorithm, whereas in the second test problem, a total savings as high as 20% was achieved. Essentially all of this improvement occurred only after the initial transient in the solution was overcome. However, in the second test problem, a significant improvement in the computational performance of the standard forward/backward VLGS algorithm was noted when overcoming the initial transient simply by converting from the use of conserved variables to primitive variables in the spatial discretization and linearization of all terms.

An Anti-Spatial-Aliasing Filter for Explicit Modeling and Imaging in Inhomogeneous Media. Alvin K. Benson. *Brigham Young University, Provo, Utah 84602, USA.*

An explicit finite difference solution to the scalar wave equation in isotropic, inhomogeneous media is completed by filtering out nonphysical contributions to the data. This digital, anti-spatial-aliasing filter and some associated limits on angular frequency are determined. The filter is a projection operator determined from a constrained least-squares fit and can be implemented in the computer algorithm at either of two places. Furthermor, the filter should be applicable to *any* explicit finite difference solution to the wave equation. Unlike a standard dip filter, this filter is computationally flexible, efficient, and necessary in inhomogeneous media with rapid lateral and vertical velocity changes.

Numerical Solution of the Steady Stokes Equations. Eric Yu Tau. *Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720, USA.*

In this paper we present a fast numerical technique for finding solutions of the steady-state Stokes equations on both two- and three-dimensional domains. We implement the method on a special staggered grid for a rectangular (cubic) domain and obtain a solution in an order of $O(N \log N)$ operations for both two- and three-dimensional cases, where N is the number of grid points in the domain. The main idea is to derive from the Stokes equations an equation for the pressure p, $Ap = b$, where the matrix A is semi-positive definite and very-well conditioned on the orthogonal complement of its null space.

Box-Counting Algorithm and Dimensional Analysis of a Pulsar. F. H. Ling and G. Schmidt. *Stevens Institute of Technology, Hoboken, New Jersey 07030, USA.*

It is argued that the use of the box-counting algorithm to calculate the correlation dimension is a better choice than the Grassberger–Procaccia (correlation integral) algorithm for dealing with an experimental data set. This is illuminated by treating three classical examples: the logistic map, the Hénon map and the Lorenz equation. The intensity data of a pulsar is also treated which is revealed to have a least embedding dimension of 14 and the correlation dimension of about 4.5.

The Inclusion of Collisional Effects in the Splitting Scheme. L. Demeio. *Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061, USA.*

An algorithm is given for the numerical solution of the Boltzmann equation for a one-dimensional unmagnetized plasma with immobile ions, in which collisional effects are described by the Bhatnagar–Gross–Krook (BGK) model. The algorithm is a straightforward generalization of the splitting scheme, which solves the one-dimensional Vlasov–Poisson system. The accuracy of the splitting scheme to second order in Δt is preserved.